Worksheet 9

Date: 11/03/2021

Name:

Division Algorithm and Primes

THEOREM 1 (The Division Algorithm)**.** *For positive integers a and b, there exist unique integers q and r such that*

$$b = aq + r \quad 0 \leq r < a$$

PROPOSITION 2. *If $a, b \in \mathbb{Z}$ and $d = hcf(a, b)$, then there are integers s and t such that*

$$d = sa + tb.$$

PROPOSITION 3. *Let a and b be positive integers. If $b = aq + r$ for some integers q and r, then* $\gcd(a, b) = \gcd(r, a)$.

What is the Euclidean algorithm and Division algorithm? This is best explained by an example. Compute

Example 4. $hcf(2880, 504)$

THEOREM 5. *Let a and b be integers, not both zero. Then a and b are relatively prime if and only if there exist integers x and y such that $1 = ax + by$.*

THEOREM 6 (Euclid's Lemma)**.** *If $a|bc$, with $(a,b)=1$, then $a|c$.*

Problems

1. Show for any integer k, $(9k+4, 2k+1) = 1$

2. If $(a,b)=1$, then $(a,b^n)=1$ for all positive integers.

3. If n is composite then n has a prime factor p such that $p \leq \sqrt{n}$

4. Suppose $a, b \in \mathbb{Z}, hcf(a,b) = d$. Prove $hcf(\frac{a}{d}, \frac{b}{d}) = 1$.